IMPACT: International Journal of Research in
Engineering & Technology (IMPACT: IJRET)
Vol. 1, Issue 3, Aug 2013, 25-34

COMPARISON OF BIT ERROR RATE IN OFDM SYSTEM BY USING MMSE EQUALIZER

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ABSTRACT

This paper compares and improves the bit error rate performance with the minimum mean squared error (MMSE) equalizer applied to wireless multiple antenna systems (MIMO) by using Orthogonal Frequency Division Multiplexing (OFDM). In Spite of much prior work on this subject, This reveal several new and surprising analytical results in terms of signal-to-noise ratio (SNR) by changing the OFDM parameters.

KEYWORDS: MMSE, OFDM, BPSK, DFT, ISI

INTRODUCTION

Consider the complex baseband model for wireless multi-input multi-output (MIMO) channel with N_t transmit antennas and N_r receiver antennas

$$Y = Hx + z, \tag{1}$$

where Y is the received signal and H is a Rayleigh fading channel with independent, identically distributed (i.i.d) [4,5], circularly symmetric standard complex Gaussian entries, denoted as $h_{ij} \sim N_r(0, 1)$ for $1 \leq i \leq N_t$; $1 \leq j \leq Nr$. We assume that the number of receive antennas is no less than the number of transmit antennas ($N_t \geq N_r$) [1]. We also assume that the *nr* data sub streams have uniform power, i.e.,$x \in \sigma^{N_r \times 1}$has covariance matrix $E[xx^*] = \sigma_x^2 I_{N_r}$, where E[.] stands for the expected value, $(.)^*$ is the conjugate transpose, and I_{N_r} is an $N_t \times N_r$ identity matrix. The white Gaussian noise $z \sim N_r(0, \sigma_z^2 I_{N_r})$ is also circularly symmetric. The input signal-to-noise ratio (SNR) is defined as [2]

$$\mathrm{SNR} = \frac{\sigma_x^2}{\sigma_z^2} \tag{2}$$

In this paper, we compare the bit error rate performance with respect to the Orthogonal Frequency Division Multiplexing (OFDM) parameters with the minimum mean squared error (MMSE) equalizer applied to the channel given in (1). The linear MMSE equalizer is classic functional blocks and is ubiquitous in digital communications [3]. This is also the building blocks of more advanced communication schemes such as the decision feedback equalizer (DFE).

ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

Orthogonality and OFDM

If the dot product of two deterministic signals is equal to zero, these signals are said to be orthogonal to each other. Orthogonality can also be viewed from the standpoint of stochastic processes. If two random processes are uncorrelated, then they are orthogonal. The random nature of signals in a communications system, this probabilistic view of orthogonality provides an intuitive understanding of the implications of orthogonality in OFDM. OFDM is implemented in practice using the discrete Fourier transform (DFT). As we know that the sinusoids of the DFT(Descrete Fourier Transform) form an orthogonal basis set, and a signal in the vector space of the DFT can be represented as a linear combination of the orthogonal sinusoids. One view of the DFT is that the transform essentially correlates its input signal with each of the sinusoidal basis functions. If the input signal has some energy at a certain frequency, there will be a peak in the correlation of the input signal and the basis sinusoid that is at that corresponding frequency. This transform is used at the OFDM transmitter to map an input signal onto a set of orthogonal sub carriers, i.e., the orthogonal basis functions of the DFT. Similarly, the transform is used again at the OFDM receiver to process the received sub carriers. The signals from the sub carriers are then combined to form an estimate of the source signal from the transmitter. The orthogonal and uncorrelated nature of the sub carriers is exploited in OFDM with powerful results. Since the basis functions of the DFT are uncorrelated, the correlation performed in the DFT for a given sub carrier only sees energy for that corresponding sub carrier. The energy from other sub carriers does not contribute because it is uncorrelated. This separation of signal energy is the reason that the OFDM sub-carriers spectrums can overlap without causing interference.

Mathematical Analysis

Mathematically, each carrier can be described as a complex wave in OFDM system

$$S_c\,(t) = A_c\,(t)e^{[j(\omega_c(t)+\phi_c(t)]} \tag{3}$$

The real signal is the real part of $S_c(t)$. A_c (t) and $\Phi_c(t)$, the amplitude and phase of the carrier, can vary on a symbol by symbol basis. The values of the parameters are constant over the symbol duration period t. OFDM consists of many carriers. Thus the complex signal $S_s(t)$ is represented by

$$S_s(t)=\frac{1}{N}\sum_{n=0}^{N-1} A_N e^{[j(\omega_n(t)+\phi_n(t)]} \tag{4}$$

where $\omega_n = \omega_o + n\Delta\omega$

This is of course a continuous signal. If we consider the waveforms of each component of the signal over one symbol period, then the variables $A_c(t)$ and φ_c (t) take on fixed values,

Which depend on the frequency of that particular carrier, and so can be rewritten as

$\Phi_n(t)= \Phi_n$, $A_n(t)=A_n$

If the signal is sampled using a sampling frequency of $\frac{1}{T}$, then the resulting signal is represented by

$$S_s(kT)=\sum_{n=0}^{N-1} A_N e^{[j(\omega_0+n\Delta\omega)kT+\phi_n(t)]} \tag{5}$$

At this point, we have restricted the time over which we analyze the signal to N samples. It is convenient to sample over the period of one data symbol. Thus we have a relationship: t=NT If we now simplify Eq.(5), without a loss of generality by letting ω_0=0, then the signal becomes

$$S_s(kT)=\sum_{n=0}^{N-1} A_N e^{[j(n\Delta\omega)kT+\phi_n(t)]} \tag{6}$$

Now equation (6) can be compared with the general form of the inverse Fourier transform

$$g(kT)=\frac{1}{N}\sum_{n=0}^{N-1}.G(\frac{n}{NT})e^{\frac{2\pi}{N}kn} \tag{7}$$

In Eq.(7) the function $A_n e^{j\Phi n}$ is no more than a definition of the signal in the sampled frequency domain, and S(kT) is the time domain representation. Eqns (6) and (7) are equivalent if:

$$\Delta\ f = \frac{\Delta\omega}{2\pi} = \frac{1}{NT} = \frac{1}{\tau}$$

This is the same condition that was required for orthogonality Thus, one consequence of maintaining orthogonality is that the OFDM signal can be defined by using Fourier transform procedures.

OFDM Generation and Reception

Figure (1) shows the block diagram of a typical OFDM transceiver [2]. The transmitter section converts digital data to be transmitted, into a mapping of subcarrier amplitude and phase. It then transforms this spectral representation of the data into the time domain using an Inverse Discrete Fourier Transform (IDFT). The Inverse Fast Fourier Transform (IFFT) performs the same operations as an IDFT, except that it is much more computationally efficiency, and so is used in all practical systems. In order to transmit the OFDM signal the calculated time domain signal is then mixed up to the required frequency.

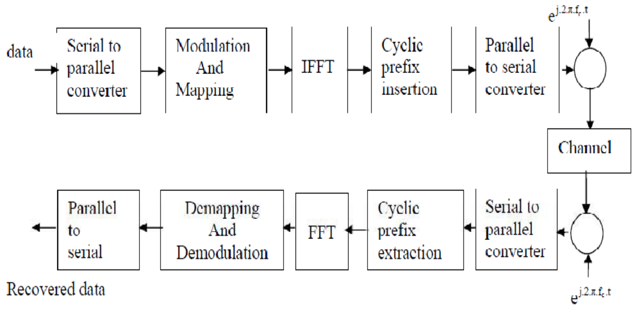

Figure 1: Block Diagram of a Basic OFDM Transceiver

The receiver performs the reverse operation of the transmitter, mixing the RF signal to base band for processing, then using a Fast Fourier Transform (FFT) to analyze the signal in the frequency domain [3]. The amplitude and phase of the sub carriers is then picked out and converted back to digital data. The IFFT and the FFT are complementary function and the most appropriate term depends on whether the signal is being received or generated. In cases where the signal is independent of this distinction then the term FFT and IFFT is used interchangeably.

Serial to Parallel Conversion

Data to be transmitted is typically in the form of a serial data stream. In OFDM, each symbol typically transmits 40 - 4000 bits, and so a serial to parallel conversion stage is needed to convert the input serial bit stream to the data to be transmitted in each OFDM symbol. The data allocated to each symbol depends on the modulation Scheme used and the number of sub carriers. At the receiver the reverse process takes place, with the data from the sub carriers being converted back to the original serial data stream. When an OFDM transmission occurs in a multipath radio environment, frequency selective fading can result in groups of sub carriers being heavily attenuated, which in turn can result in bit errors. These nulls in the frequency response of the channel can cause the information sent in neighboring carriers to be destroyed, resulting in a clustering of the bit errors in each symbol. Most Forward Error Correction (FEC) schemes tend to work more effectively if the errors are spread evenly, rather than in large clusters, and so to improve the performance most systems employ data scrambling as part of the serial to parallel conversion stage. This is implemented by randomizing the sub carrier allocation of each sequential data bit. At the receiver the reverse scrambling is used to decode the signal. This restores the original sequencing of the data bits, but spreads clusters of bit errors so that they are approximately uniformly distributed in time. This randomization of the location of the bit errors improves the performance of the FEC and the system as a whole.

Guard Period

For a given system bandwidth the symbol rate for an OFDM signal is much lower than a single carrier transmission scheme. For a single carrier BPSK modulation, the symbol rate corresponds to the bit rate of the transmission. However for OFDM the system bandwidth is broken up into nr sub carriers, resulting in a symbol rate that is nr times lower than the single carrier transmission. This low symbol rate makes OFDM naturally resistant to effects of Inter-Symbol Interference (ISI) caused by multipath propagation. Multipath propagation is caused by the radio transmission signal reflecting off objects in the propagation environment. Figure (2) shows the addition of guard period to OFDM system.

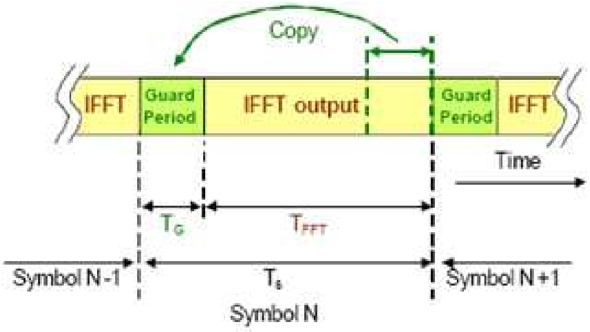

Figure 2: Addition of a Guard Period to an OFDM Signal

These multiple signals arrive at the receiver at different times due to the transmission distances being different. This spreads the symbol boundaries causing energy leakage between them. The effect of ISI on an OFDM signal can be further improved by the addition of a guard period to the start of each symbol. This guard period is a cyclic copy that extends the length of the symbol waveform. Each sub carrier, in the data section of the symbol, (i.e. the OFDM symbol with no guard period added, which is equal to the length of the IFFT size used to generate the signal) has an integer number of cycles. Because of this, placing copies of the symbol end-to-end results in a continuous signal, with no discontinuities at the joins. Thus by copying the end of a symbol and appending this to the start results in a longer symbol time. The total length of the symbol is T_s=T_G + T_{FFT}, where T_s is the total length of the symbol in samples, T_G is the length of the guard period in samples, and T_{FFT} is the size of the IFFT used to generate the OFDM signal. In addition to protecting the OFDM from ISI, the guard period also provides protection against time-offset errors in the receiver [6,7,8].

Protection against Time Offset

To decode the OFDM signal the receiver has to take the FFT of each received symbol, to work out the phase and amplitude of the sub carriers. For an OFDM system that has the same sample rate for both the transmitter and receiver, it must use The same FFT size at both the receiver and transmitted signal in order to maintain sub carrier orthogonality. Each received symbol has T_G + T_{FFT} samples due to the added guard period. The receiver only needs T_{FFT} samples of the received symbol to decode the signal. The remaining T_G samples are redundant and are not needed. For an ideal channel with no delay spread the receiver can pick any time offset, up to the length of the guard period, and still get the correct number of samples, without crossing a symbol boundary. Because of the cyclic nature of the guard period changing the time offset simply results in a phase rotation of all the sub carriers in the signal. The amount of this phase rotation is proportional to the sub carrier frequency, with a sub carrier at the Nyquist frequency changing by 180° for each sample time offset. Provided the time offset is held constant from symbol to symbol, the phase rotation due to a time offset can be removed out as part of the channel equalization. In multipath environments ISI reduces the effective length of the guard period leading to a corresponding reduction in the allowable time offset error. The addition of guard period removes most of the effects of ISI. However in practice, multipath components tend to decay slowly with time, resulting in some ISI even when a relatively long guard period is used [9,10].

Guard Period Overhead and Sub Carrier Spacing

Adding a guard period lowers the symbol rate, however it does not affect the sub carrier spacing seen by the receiver. The sub carrier spacing is determined by the sample rate and the FFT size used to analyze the received signal.

$$\Delta f = \frac{F_s}{N_{FFT}}$$

In above equation, Δf is the sub carrier spacing in Hz, Fs is the sample rate in Hz, and N_{FFT} is the size of the FFT. The guard period adds time overhead, decreasing the overall spectral efficiency of the system.

MMSE EQUALIZER

In this paper, The MMSE Equalizer algorithm is used at the receiver of the OFDM system to improve the bit error rate. Consider the MIMO channel model given in Eqn.(1) where the *N* data sub streams are mixed by the channel matrix. The MMSE equalizer can be applied to decouple the *N* sub streams. The MMSE equalization matrices are

$$W_{mmse} = \left(H^*H + \frac{1}{SNR} I \right)^{-1} H^*$$

Left multiplying the received signal vector y by W_{mmse} , we obtain *nr* decoupled sub streams with output SNRs

$$\rho_{mmse,n} = \frac{SNR}{\left[\left(H^*H + \frac{1}{SNR} I\right)^{-1}\right]_{nn}} - 1, 1 \le n \le N_r$$

Here $[.]_{nn}$ denotes the n th diagonal element. Denote h_n the nth column of H and H_n the submatrix obtained by striking h_n out of H. Hence

$$[(H^*H)]_{nn} = h_n^* h_n - h_n^* H_n (H_n^* H_n)^{-1} H_n^* h_n$$

That

$$\rho_{mmse,n} = \frac{SNR}{\left[\left(h_n^* h_n - h_n^* H_n (H_n^* H_n)^{-1} H_n^* h_n + \frac{1}{SNR} I \right)^{-1} \right]_{nn}}$$

$$= \left[\left(h_n^* h_n - h_n^* H_n \left(H_n^* H_n + \frac{1}{SNR} I \right)^{-1} H_n^* h_n \right)^{-1} \right] SNR,$$

$1 \le n \le N_r$.

MIMO WITH MMSE EQUALIZER

In a 2×2 MIMO channel, consider that we have a transmission sequence, for example $\{x_1, x_2, x_3, x_4\}$, In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot, x_3 and so on. However, as we now have 2 transmit antennas, we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second antenna; send x_5 and x_6 in the third time slot and so on. Notice that as we are grouping two symbols and sending them in one time slot, we need only $\frac{n}{2}$ time slots to complete the transmission hence the data rate is doubled. This forms the simple explanation of a probable MIMO transmission scheme with 2 transmit antennas and 2 receive antennas.

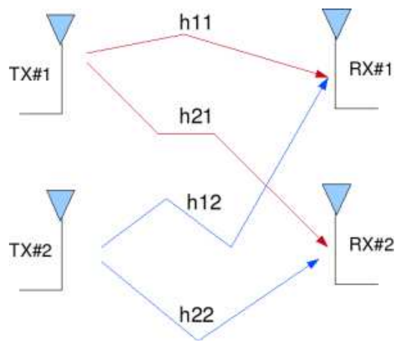

Figure 3: 2 Transmit and 2 Receive (2×2) MIMO Channel

In this paper, we are assuming that the channel is flat fading, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication and the channel experience by each transmit antenna is independent from the channel experienced by other transmit antennas. For the i^{th} transmit antenna to j^{th} receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number $h_{i,j}$.

As the channel under consideration is a Rayleigh channel, the real and imaginary parts of $h_{i,j}$ are Gaussian distributed having mean $\mu_{hi,j}$ and variance $\sigma^2_{hi,j} = \frac{1}{2}$. The channel experienced between each transmit to the receive antenna is independent and randomly varying in time. On the receive antenna, the noise n has the Gaussian probability density function with

$$P(n) = \frac{1}{\sqrt{2}\pi\sigma^2} e^{\frac{-(n-\mu)^2}{2\sigma^2}} \text{ with } \mu = 0 \text{ and } \sigma^2 = \frac{N}{2}$$

The channel $h_{i,j}$ is known at the receiver. Let us understand the mathematics for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$Y_1 = h_{1,1} x_1 + h_{1,2} x_2 + n_1 = [\, h_{1,1}\ h_{1,2}\,]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the second receive antenna is,

$$Y_2 = h_{2,1} x_1 + h_{2,2} x_2 + n_2 = [\, h_{2,1}\ h_{2,2}\,]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where Y_1 and Y_2 are the received symbol on the first and second antenna respectively. $h_{1,1}$ is the channel from first transmit antenna to first receive antenna. $h_{1,2}$ is the channel from second transmit antenna tofirst receive antenna. $h_{2,1}$ is the channel from first transmit antenna to second receive antenna. $h_{2,2}$ is the channel from second transmit antenna to second receive antenna x_1, x_2 are the transmitted symbols and n_1 , n_2 is the noise on first and second receive antennas. We assume that the receiver knows $h_{1,1}$, $h_{1,2}$, $h_{2,1}$ and $h_{2,2}$. The receiver also knows y_1 and y_2. Hence

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Equivalently,

$$Y = Hx + n$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which minimizes the criterion,

$$E\{ [\, Wy - x\,][\, Wy - x]^H \}$$

Solving,

$$W = [H^H H + N_0 I]^{-1} H^H$$

NUMERICAL EXAMPLE USING MATLAB

Case 1

In this paper, the simulation is being done by using the MATLAB and the step by step procedure to find the results of bit error rate with the prescribed concepts is being described. Figure (4) shows the bit error rate with MMSE equalizer, BPSK modulation and 2×2 MIMO but without OFDM system. It is clearly seen from the simulation result that, in increase SNR the bit error get decrease.

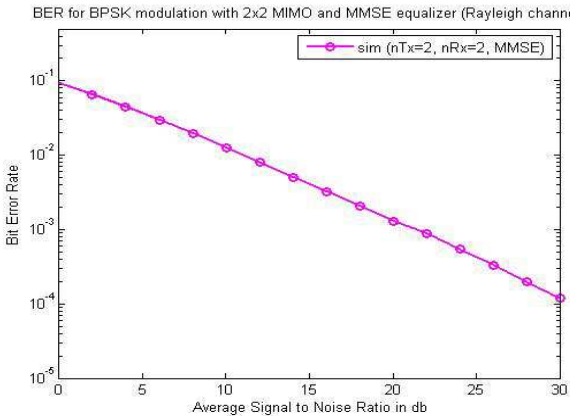

Figure 4: BER with MIMO and MMSE without OFDM

Case 2

Figure (5) represents the simulation result of bit error rate reduction in OFDM system using MMSE equalizer and the parameter are selected as Number of transmitting antennas(N_t) = 3, Number of receiving antennas(N_r) = 2, Frame length(FL) = 50,Channel Order(L) = 5, Cyclic Prefix Length (CP) = 16 and FFT Size (FS) is varied from 16 to 512 and then corresponding results are compared. The lowest bit error rate is found at FFT Size = 512.

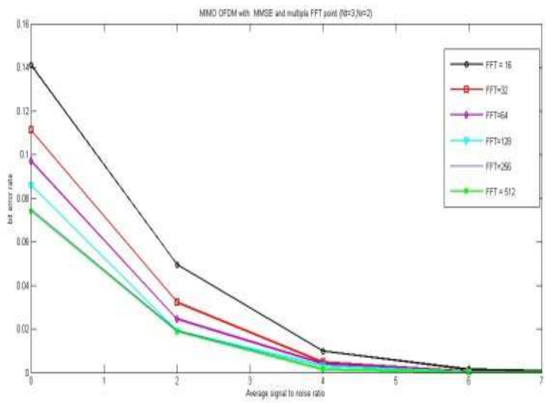

Figure 5: MIMO OFDM with MMSE and Multiple FFT Point (N_t=3, $N_{r=}$2)

Case 3

Figure (6) represents the simulation result of bit error rate reduction in OFDM system using MMSE equalizer and the parameter are selected as N_t = 2, N_r = 2, FL = 50, L=5, CP= 16 and FS is varied from 16 to 512 and then corresponding results are compared. The lowest bit error rate is found at FS= 512 and at higher value of FS after 512, BER does not get remarkable reduction.

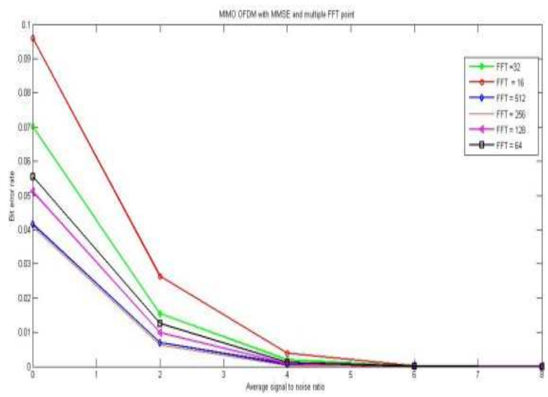

Figure 6: MIMO OFDM with MMSE and Multi Point FFT (N_t =2=N_r)

Case 4

Figure (7) represents the simulation result of bit error rate reduction in OFDM system using MMSE equalizer and the parameter are selected as $N_t = N_r = 3$, FL = 50, L=5, CP= 16 and FS is varied from 16 to 512 and then corresponding results are compared. The lowest bit error rate is found at FS= 512 and at higher value of FS after 512, BER does not get remarkable reduction.

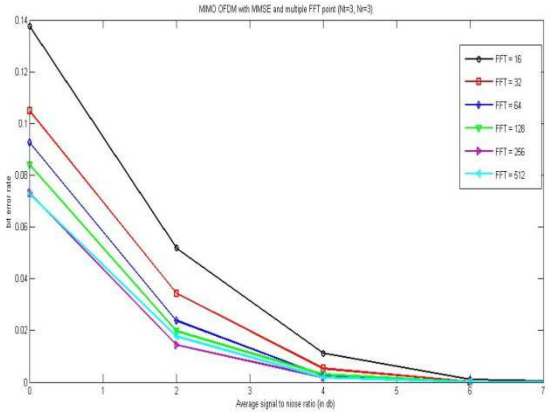

Figure 7: MIMO OFDM with MMSE and Multi Point FFT (N_t=3=N_r)

Case 5

Figure (8) represents the simulation result of bit error rate reduction in OFDM system using MMSE equalizer and the parameter are selected as $N_t = N_r = 4$, FL = 50, L=5, CP= 16 and FS is varied from 16 to 512 and then corresponding results are compared.

The lowest bit error rate is found at FS= 512 and at higher value of FS after 512, BER does not get remarkable reduction.

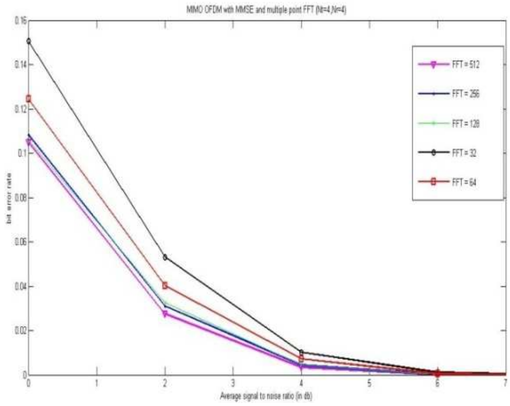

Figure 8: MIMO OFDM with MMSE and Multi Point FFT (N_t=4=N_r)

Case 6

Figure (9) represents the simulation result of bit error rate reduction in OFDM system using MMSE equalizer and the parameter are selected as $N_t = N_r = 5$, FL = 50, L=5, CP= 16 and FS is varied from 16 to 512 and then corresponding results are compared.

The lowest bit error rate is found at FS= 512 and at higher value of FS after 512, BER does not get remarkable reduction.

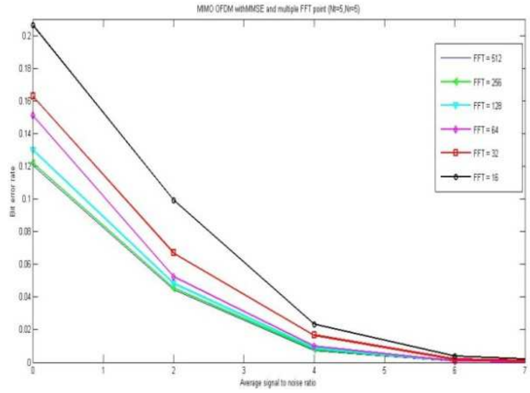

Figure 9: MIMO OFDM with MMSE and Multi Point FFT (N_t=5=N_r)

Case 7

Figure (10) represents the simulation result of bit error rate reduction in OFDM system using MMSE equalizer. The FL, Number of antennas, and CP are varied at the FS=256 and a comparison is made as shown in figure.

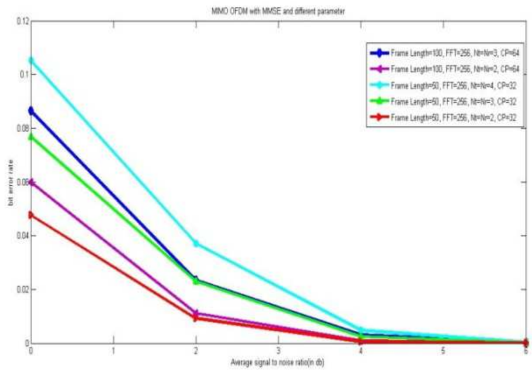

Figure 10: MIMO OFDM with MMSE and Different Parameters

CONCLUSIONS

Multi-path propagation leads to ISI in received signal. To eliminate ISI, we need to equalize the received signal. MMSE equalizer is employed in time domain for this purpose. The BER performance improves significantly using these equalizers, but still ISI is not eliminated completely. The BER is approaching theoretical limits. Work can be done to further improve BER and reliability. Work can be further extended in more general Nakagami-*m* channels. Also performance can be investigated in other novel equalizers.

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